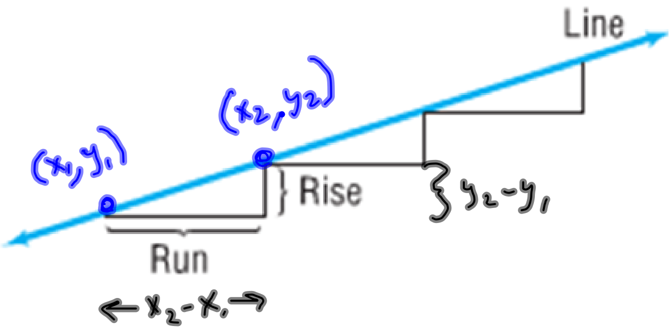
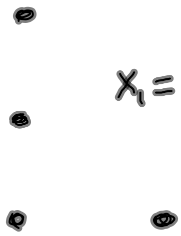
1 Calculate and Interpret the Slope of a Line

Figure 24





$x_1 = x_2$

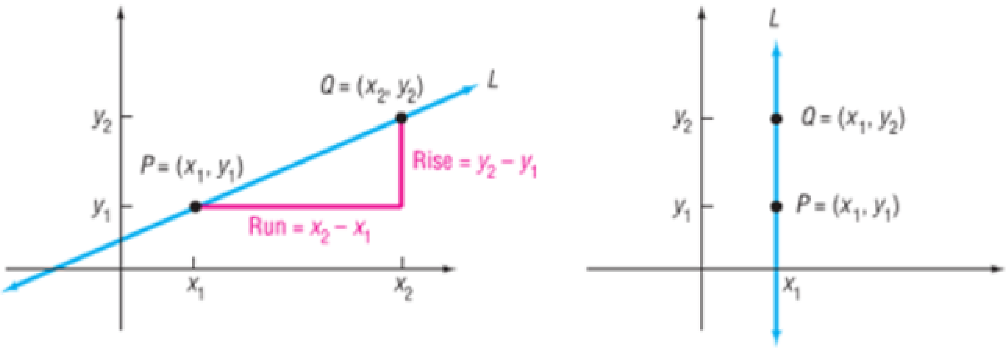
$y_1 = y_2$

Let $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ be two distinct points. If $x_1 \neq x_2$, the **slope** m of the nonvertical line L containing P and Q is defined by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \underline{x_1 \neq x_2} \qquad (1)$$

If $x_1 = x_2$, L is a **vertical line** and the slope m of L is **undefined** (since this results in division by 0).

Figure 25

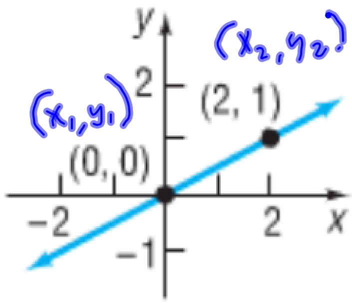


(a) Slope of L is $m = \frac{y_2 - y_1}{x_2 - x_1}$ (b) Slope is undefined; L is vertical

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Rise}}{\text{Run}} \qquad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\Delta y}{\Delta x}$$

In Problems 11–14, (a) find the slope of the line and (b) interpret the slope.

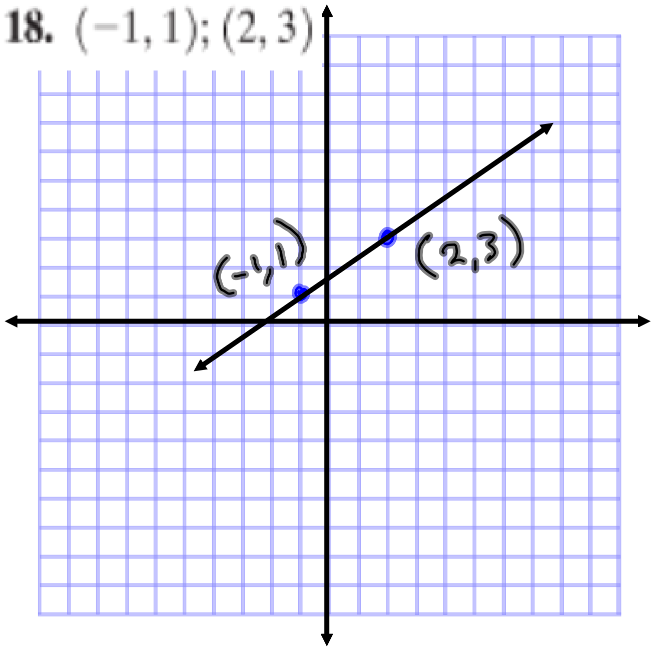
11.

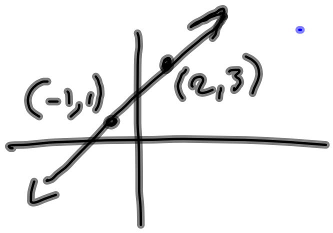


$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{2 - 0} = \frac{1}{2}$$

In Problems 15–22, plot each pair of points and determine the slope of the line containing them. Graph the line.

18. $(-1, 1); (2, 3)$

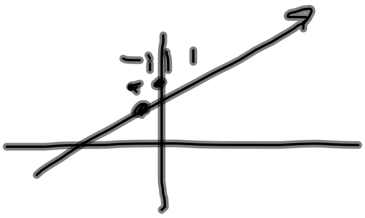




For every ___3___ change in x, y will change by ___2___ units" is the way the book would say it, and is the best way to think of it, if you were trying to graph it using just one of the points and the slope of the line. But it's also useful to always think of the x value increasing by *one*, so that you say "y changes ___2/3___ unit(s) per *unit change* in x.

18. $(-1, 1); (2, 3)$
$(x_1, y_1)\ (x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{2-(-1)} = \frac{2}{3}$$

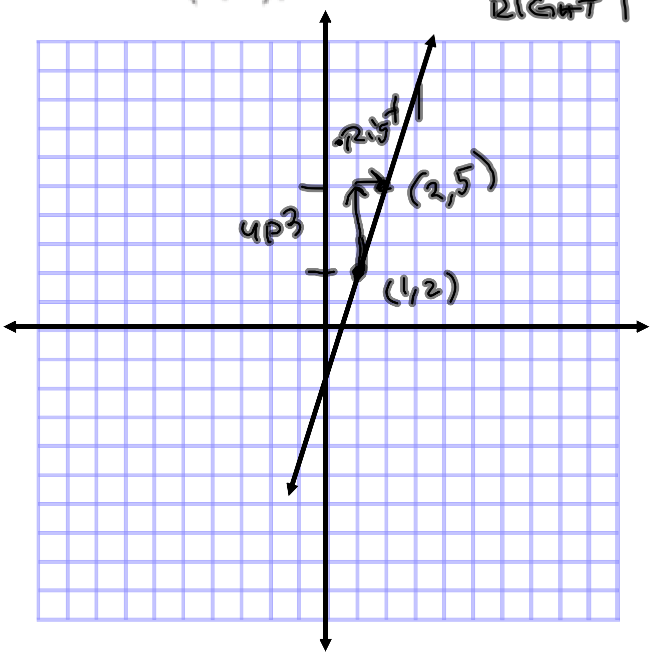


$\frac{\text{up } 2}{\text{right} + 3}$ OR $\frac{\text{up } 2/3}{\text{RIGHT } 1}$

2 Graph Lines Given a Point and the Slope

In Problems 23–30, graph the line containing the point P and having slope m.

23. $P = (1, 2); m = 3 = \frac{\text{up } 3}{\text{RIGHT } 1}$



3 Find the Equation of a Vertical Line thru...

$(-7, 18)$ $x = -7$

$(5, -372, 481)$ $x = 5$

4 Use the Point–Slope Form of a Line; Identify Horizontal Lines

Point–Slope Form of an Equation of a Line

An equation of a nonvertical line with slope m that contains the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \quad (2)$$

$+y_1 = \quad +y_1$

$y = m(x-x_1) + y_1$ I like it.

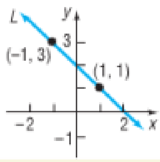
An equation of a nonvertical line with slope m that contains the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \qquad (2)$$

In #s 37 – 44, find an equation of the line L.

There are 3 acceptable types of answers, and an infinite number of variations in 2 of the 3 types. Only the slope-intercept form is unique. Use the one you prefer. Your teacher is obliged to check every answer with care (so be gentle!). ***The point-slope form is the easiest one to get,*** *when method is left to the student.*

39.



Slope–Intercept Form of an Equation of a Line

An equation of a line L with slope m and y-intercept b is

In #s 37 - 44, find an equation of the line L.

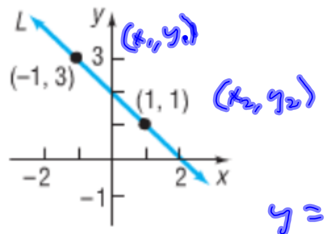
There are 3 acceptable types of answers, and an infinite number of variations in 2 of the 3 types. Only the slope-intercept form is unique. Use the one you prefer. Your teacher is obliged to check every answer with care (so be gentle!). ***The point-slope form is the easiest one to get,*** *when method is left to the student*

Point-Slope $y=m(x-x_1)+y_1$

Slope Intercept $y=mx+b$

General $Ax+By=C$, A,B,C constant.

39.



$m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{1-3}{1-(-1)}=\dfrac{-2}{2}=-1$

Two ways!

$(x_0, y_0) = (1, 1)$

$y=mx+b$

$y=-1x+b$

$1=-1\cdot 1+b$

$1=-1+b$

$2=b$

$\boxed{y=-x+2}$

$y=m(x-x_1)+y_1$

$\boxed{y=-1(x-1)+1}$

$y-y_1=m(x-x_1)$

$y-1=-1(x-1)$

$y=-1(x+1)+3$

$y-3=-1(x+1)$

Slope–Intercept Form of an Equation of a Line

An equation of a line L with slope m and y-intercept b is

$$y = mx + b \quad (3)$$

The equation of a line L is in **general form*** when it is written as

$$Ax + By = C \quad (4)$$

where A, B, and C are real numbers and A and B are not both 0.

General Form is great for graphing, because you can "read" the intercepts from it. Also, systems of equations.

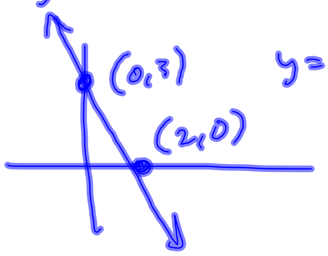
$3x+2y=6$

$x=0 \Rightarrow 2y=6$

$y=3 \rightarrow (0,3)$

$y=0 \Rightarrow 3x=6$

$x=2 \rightarrow (2,0)$



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#s 45 - 70, find an equation of the line with the given properties. Express your answer using either the general form or the slope-intercept form.

47. Slope $= -\frac{2}{3}$; containing the point $\overset{(x_1, y_1)}{(1, -1)}$

$y = m(x - x_1) + y_1$

$y = -\frac{2}{3}(x-1) - 1$

$= -\frac{2}{3}x + \frac{2}{3} - 1$

$\boxed{y = -\frac{2}{3}x - \frac{1}{3}}$

$+\frac{2}{3}x = +\frac{2}{3}x$

$\frac{2}{3}x + y = -\frac{1}{3}$ ✓

$2x + 3x = -1$ General

High School

$y = mx + b$

$-1 = (-\frac{2}{3})(1) + b$

$-1 = -\frac{2}{3} + b$

$+\frac{2}{3} = +\frac{2}{3}$

$-\frac{1}{3} = b$

$y = -\frac{2}{3}x - \frac{1}{3}$

49. Containing the points $(1, 3)$ and $(-1, 2)$

(x_1, y_1) (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-3}{-1-1} = \frac{-1}{-2} = \frac{1}{2}$$

$$y = m(x - x_1) + y_1$$

$$y = \frac{1}{2}(x-1) + 3$$

$$y = \frac{1}{2}x - \frac{1}{2} + 3$$

$$y = \frac{1}{2}x + \frac{5}{2}$$

$$-\frac{1}{2} + 3$$

$$= -\frac{1}{2} + \frac{6}{2} = \frac{5}{2}$$

$$(2)\left(\frac{1}{2}\right) = 1$$

$$2\frac{1}{2}$$

51. Slope = −3; y-intercept = 3 → (0,3)

Ow!

b=3

$y = mx + b$

$y = -3x + 3$

I don't like the book using "3" as a y-intercept. I want an ordered pair!!!

Equation of a Horizontal Line

A horizontal line is given by an equation of the form

$$\boxed{y = b}$$

where b is the y-intercept.

Is a function

$y = 7$

55. Slope undefined; containing the point (2, 4) $x = 2$ Not a function.

57. Horizontal; containing the point (−3, 2) $y = 2$

Criterion for Parallel Lines

Two nonvertical lines are parallel if and only if their slopes are equal and they have different y-intercepts.

63. Parallel to the line $x = 5$; containing the point (4, 2)

$\frac{y_2 - y_1}{5 - 5} = \frac{y_2 - y_1}{0}$ undefined $\boxed{x = 4}$

$\parallel$ to $2x - 3y = 6$ thru $(-7, 11)$

$m = ?$

$Ax + By = C$

$m = -\frac{A}{B}$

$2x - 3y = 6$

$-3y = -2x + 6$

$y = \frac{-2}{-3}x + \frac{6}{-3}$

$= \frac{2}{3}x - 2$

$m = \frac{2}{3}$

$A = 2, B = -3$ ✓

$-\frac{A}{B} = \frac{2}{3}$ ✓

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Criterion for Perpendicular Lines

Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

m_1, m_2 & the lines are perp.

$m_1 \cdot m_2 = -1$

$m_2 = -\frac{1}{m_1}$ "How we see it"

" m perp " $m_{\perp}$

65. Perpendicular to the line $y = \frac{1}{2}x + 4$; containing the point $\underset{}{\overset{(x_1, y_1)}{(1, -2)}}$

$m = \frac{1}{2} \Longrightarrow m_{\perp} = -2$

$y = m(x - x_1) + y_1$

$\boxed{y = -2(x-1) + 2}$

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68. Perpendicular to the line $x - 2y = -5$; containing the point $(0, 4)$

(x_1, y_1)

$m = \frac{1}{2} \Rightarrow m_{\perp} = -2$

$y = m(x - x_1) + y_1$

$\boxed{y = -2(x - 0) + 4}$

$y = -2x + 4$

F.4 - Circles

Find the center and radius and sketch the graph of the circle (show center and 4 antipodal points) given by

$$x^2 - 4x + y^2 + 14y = -28$$

$(x-h)^2 + (y-k)^2 = r^2$ *

(h,k) is the center

(x,y) is any point on the circle.

$1x^2 - 4x$ ___ $+ y^2 + 14y$ ___ $= -28$

$\frac{4}{2} = 2$ $\qquad \frac{14}{2} = 7 \to 7^2$

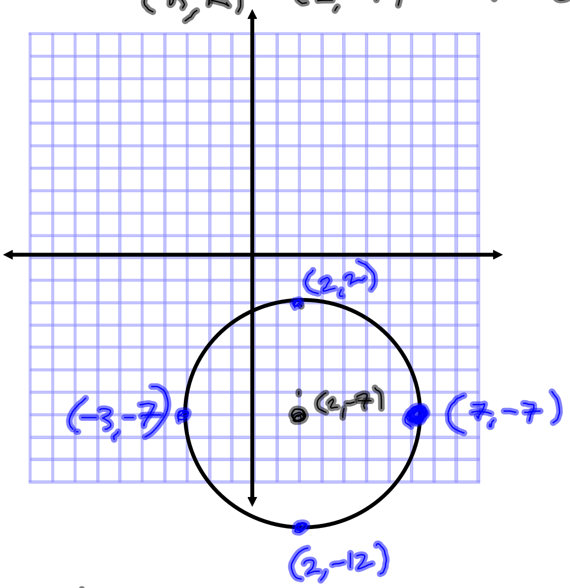
$x^2 - 4x + 2^2 \quad + y^2 + 14y + 7^2 \quad = -28 + 4 + 49$

$x^2 + 2bx + b^2$

$(x+b)^2$

$(x-2)^2 + (y+7)^2 = 25$

$(h,k) = (2,-7) \qquad r = 5$



Divide ~~$\frac{2}{x-5x}$~~ $x^4 - 2x^2 + 5x - 11$ by $x+1$

-1 | 1 $\;$ 0 $\;$ -2 $\;$ 5 $\;$ -11

There's no x^3 term, idiot

-1

-1	1	0	-2	5	-11
		-1	3	-8	
	1	-3	8	-19	

This says $x^4 - 2x^2 + 5x - 11$

$= x^3 - 3x^2 +$